FILLING AND EMPTYING OF VESSELS OF LIMITED CAPACITY WITH COMPRESSED GAS IN THE CASE OF CONSTANT AND VARIABLE VOLUME

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Design relation and graphs are obtained for calculating the filling and emptying of vessels of limited capacity by compressed gas in the case of constant and variable volume.

Filling of vessels of limited capacity and constant volume. The solution of the filling problem for a vessel of limited capacity reduces to finding the pressure-time relationship. The other parameters of the gas, connected with the pressure and the volume of the vessel, can be found from known relations.

To solve the problem we use expressions for the mass flow per second in subcritical and supercritical flow conditions.

For subcritical flow

$$G_{\text{subcr}} = \alpha f \left\{ \frac{2gn}{n-1} \frac{p_0}{v_0} \left[\left(\frac{p_i}{p_0} \right)^{\frac{2}{n}} - \left(\frac{p_i}{p_0} \right)^{\frac{n+1}{n}} \right] \right\}^{1/2} \tag{1}$$

when

$$\frac{p_i}{p_0} < \beta_{\text{cr}} = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}}$$

the flow rate is given by the formula for supercritical flow

$$G_{\text{supercr}} = \alpha f \left[\frac{2gn}{n+1} \frac{p_0}{v_0} \left(\frac{2}{n+1} \right)^{\frac{2}{n-1}} \right]^{1/2}. \tag{2}$$

Supercritical filling. The weight of gas entering the vessel in time dt will be

$$dG = G_{\text{sec}} dt. \tag{3}$$

For supercritical flow the expression has the form

$$dG = \alpha f \left[\frac{2gn}{n+1} \frac{p_0}{v_0} \left(\frac{2}{n+1} \right)^{\frac{2}{n-1}} \right]^{1/2} dt. \tag{4}$$

The left-hand side of Eq. (4) can be written as:

$$G = V/v_i, \ dG = d(V/v_i). \tag{5}$$

The variation of the specific volume v_i depends on the conditions of heat exchange between the vessel and the external medium and on the amount of gas entering the vessel.

We will consider two cases: adiabatic variation of the state of the gas in the vessel, where the heat loss from the gas during filling is small and can be neglected, and the isothermal process, where heat transfer is so rapid that the gas temperature always remains constant.

In the latter case, according to [4],

$$d(1/v_i) = dp_i/k'RT_0. \tag{6}$$

In the case of supercritical flow Eq. (4) combined with (6) takes the form

$$dG = \frac{V}{k'RT_0} dp_i = \alpha f p_0 \sqrt{\frac{2gn}{(n+1)RT_0}} \beta_{\text{cr}}^{2/n} dt. \tag{7}$$

We introduce the dimensionless quantities:

$$\beta_i = \frac{p_i}{p_0}, \quad \tau = \frac{t}{\tau_0}, \quad \tau_0 = V\left[\alpha f_{\max}\left(\frac{2gnRT_0}{n+1}\beta_{cr}^{2/n}\right)^{1/2}\right]^{-1}. \tag{8}$$

As we will see below, τ_0 is the time constant of supercritical emptying in the case of isothermal variation of the parameters of the gas in the vessel. This quantity can conveniently be taken as the unit of dimensionless time. It can be found experimentally with sufficient accuracy.

In dimensionless quantities Eq. (7) takes the form

$$d\beta = k'd\tau, \tag{9}$$

and its integral will be

$$k'\tau = \beta_2 - \beta_1. \tag{10}$$

This expression gives the time in which the value β_1 increases to β_2.

For the case of an isothermal process, k' = 1 in expression (6) and

$$\tau = \beta_2 - \beta_1. \tag{11}$$

Subcritical filling. For subcritical flow with adiabatic variation of the gas parameters Eq. (3) combined with (6) takes the form

$$dG = \alpha f\left\{\frac{2gn}{n-1}\left(\beta^{\frac{2}{n}} - \beta^{\frac{n+1}{n}}\right)p_0/v_0\right\}^{1/2} dt = \frac{Vdp_i}{k'RT_0} \tag{12}$$

or

$$\frac{\alpha fk'}{V}\sqrt{\frac{2gnRT_0}{n-1}}\,dt = \left(\beta^{\frac{2}{n}} - \beta^{\frac{n+1}{n}}\right)^{-\frac{1}{2}}d\beta. \tag{13}$$

Whence

$$k'\sqrt{\frac{n+1}{n-1}}\,\beta_{cr}^{\frac{1}{n}}\tau = \int_{\beta_1>\beta_{cr}}^{\beta_2}\left(\beta^{\frac{2}{n}} - \beta^{\frac{n+1}{n}}\right)d\beta. \tag{14}$$

The right-hand side of Eq. (14) is a binomial differential, which can be integrated, according to Chebyshev's condition, for any n.

Performing the integration, we obtain

$$k'\tau = 2\frac{\beta_{cr}^{\frac{1}{n}}}{\sqrt{n^2-1}}\left(\sqrt{1-\beta_1^{\frac{n-1}{n}}} - \sqrt{1-\beta_2^{\frac{n-1}{n}}}\right). \tag{15}$$

In the case of isothermal variation of the parameters of the gas in the vessel, k' = 1 in (6) and the integral of Eq. (13) will have the form

$$\tau = 2\frac{\beta_{cr}^{\frac{1}{n}}}{\sqrt{n^2-1}}\left(\sqrt{1-\beta_1^{\frac{n-1}{n}}} - \sqrt{1-\beta_2^{\frac{n-1}{n}}}\right). \tag{16}$$

The obtained expressions (10), (11), (15), and (16) show that in the adiabatic process the time constant of emptying is reduced by a factor of k', because $k'\tau = tk'/\tau_0$, and the increase of pressure in the vessel is accordingly more rapid. Figure 1 shows the curves plotted from expressions (10), (11), (15), and (16). From these curves the filling of the vessel can be calculated.

In the case of adiabatic processes in the vessel and on the assumption that n = k', the total filling time from β_1 =

$= 0$ to $\beta_2 = 1$ amounts to:

$$\tau_{\text{supercr}} + \tau_{\text{subcr}} = \frac{\beta_{\text{cr}}}{k'} + 2\frac{\beta_{\text{cr}}^{1/k'}}{k'\sqrt{k'^2 - 1}}\sqrt{1 - \frac{2}{k' + 1}} =$$
$$= \frac{\beta_{\text{cr}}}{k'} + 2\frac{\beta_{\text{cr}}^{1/k'}}{k'(k' + 1)} = \frac{1}{k'}\left(\beta_{\text{cr}} + 2\frac{\beta_{\text{cr}}^{1/k'}}{k' + 1}\right). \tag{17}$$

We can assume approximately that in the isothermal process

$$\tau_{\text{supercr}} + \tau_{\text{subcr}} = 2\beta_{\text{cr}}. \tag{18}$$

The value β_{cr} is determined from the index n of the process occurring in the stream of gas.

Calculations show that curves calculated for the isothermal process and for values of n from 1.01 to 1.41 differ little from one another. Hence, the main quantity affecting the course of the pressure increase curve is the index k' of the process of change of state of the gas in the vessel.

The illustrated curves correspond to the boundary values of k'. In the presence of heat transfer that does not correspond to the boundary conditions the curves of pressure increase in the vessel lie between these curves.

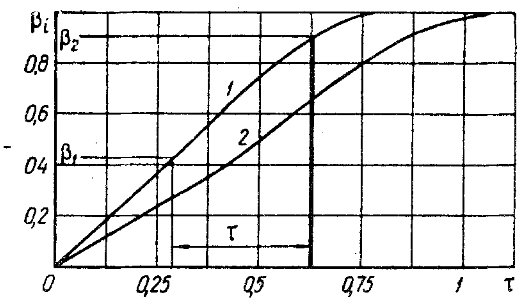

Fig. 1. Graphs for determining the filling time in the case of adiabatic and isothermal variation of the state of the gas in the vessel: 1) k' = n = 1.4; 2) k' = 1, n = 1.4.

Emptying of vessels of limited capacity and constant volume. In the process of emptying the amount of gas contained in the vessel decreases with time. Hence, the change in this amount in dt seconds will be

$$-dG = G_{\text{sec}}dt. \tag{19}$$

As before, the expressions G_{sec} are given by formulas (1) and (2), depending on the flow conditions.

The quantities p_0 and v_0 are not constant. In this case their variation is given by the equation of state, since no energy enters from the outside when the vessel is emptying.

Supercritical emptying. We substitute in Eq. (19) the value of G_{sec} for supercritical outflow and after known transformations we obtain

$$dt = -V\sqrt{\frac{1}{v_{i0} - p_{i0}}}\left(\frac{p_i}{p_{i0}}\right)^{\frac{1-3k'}{2k'}} \times$$
$$\times \frac{dp_i}{p_{i0}}\left[\alpha f k'\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\sqrt{\frac{2gn}{n+1}}\right]^{-1}. \tag{20}$$

For isothermal variation of the state of the gas in the vessel (k' = 1) we have

$$dt = -V\frac{dp_i}{p_i}\left(\alpha f \beta_{\text{cr}}^{\frac{1}{n}}\sqrt{\frac{2gn}{n+1}}\sqrt{RT_{i0}}\right)^{-1} \tag{21}$$

or

$$d\tau = -\frac{dp_i}{p_i}.$$

Integrating in the limits of variation of gas pressure, we obtain

$$p_i = p_{i0}\exp(-\tau) \quad \text{or} \quad \beta_u = \exp(-\tau), \tag{22}$$

where $\beta_u = p_i/p_{i0}$.

As mentioned above, $\tau_0 = \frac{V}{\alpha f}\left(\frac{2gnRT_{i0}}{n+1}\beta_{cr}^{\frac{2}{n}}\right)^{-1/2}$ is the constant of the exponential curve of decrease of pressure in the vessel. This value can be obtained experimentally.

In the case of outflow from the vessel into a medium with constant pressure relation (22) will be as follows:

$$\beta_i = \beta_{i0} \exp \tau, \quad (23)$$

where

$$\beta_{i0} = p_a/p_{i0}, \quad \beta_i = p_a/p_i.$$

Supercritical flow ceases as soon as β_i becomes greater than β_{cr}.

For adiabatic variation of the state of the gas in the vessel expression (20) takes the form

$$dt = V\left(\frac{p_i}{p_{i0}}\right)^{\frac{1-3k'}{2k'}}\left(\alpha f k' \beta_{cr}^{\frac{1}{n}}\sqrt{\frac{2gnRT_{i0}}{n+1}}\right)^{-1}\frac{dp_i}{p_{i0}} \quad (24)$$

or

$$d\tau = -\beta_u^{\frac{1-3k'}{2k'}} d\beta_u. \quad (25)$$

After integration we arrive at the expression

$$\tau = \frac{2}{k'-1}\left(\beta_{u_2}^{\frac{1-k'}{2k'}} - \beta_{u_1}^{\frac{1-k'}{2k'}}\right). \quad (26)$$

The values of the relative pressure β_u are determined not from the initial pressure p_{i0}, but for p_a. In this case

$$\beta_u = \beta_{i0}/\beta_i. \quad (27)$$

Figure 2 presents curves calculated from formulas (23) and (26). The curves show that for adiabatic variation of the state of the gas in the vessel emptying is a little more rapid than under isothermal conditions.

Subcritical emptying. For subcritical flow in the case of isothermal variation of the state of the gas in the vessel we obtain the following relation between pressure and time:

$$-\frac{dp_i}{p_i} \times \left[\left(\frac{p_a}{p_i}\right)^{\frac{2}{n}} - \left(\frac{p_a}{p_i}\right)^{\frac{n+1}{n}}\right]^{-\frac{1}{2}} = \frac{\alpha f\sqrt{T}}{V}\sqrt{\frac{2gnR}{n-1}}\,dt. \quad (28)$$

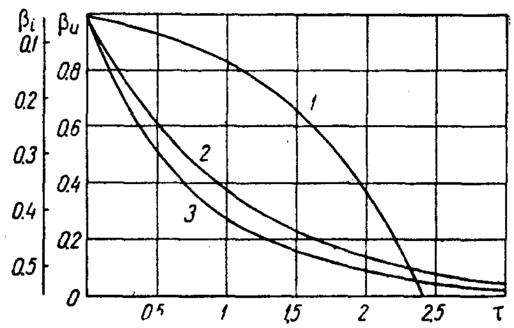

Fig. 2. Graphs for determining emptying time in the case of supercritical flow: 1) $\beta_i = \beta_{i0} \exp \tau$, $\beta_{i0} = 0.05$; 2) $\beta_u = \exp(-\tau)$, $k' = 1$, $n = 1.4$; 3) $k' = n = 1.4$; τ from (26)

Since

$$p_a/p_i = \beta_i \text{ and } dp_i = -p_a\, d\beta_i/\beta_i^2,$$
$$-dp_i/p_i = d\beta_i/\beta_i.$$

In view of these expressions Eq. (28) becomes

$$\frac{d\beta_i}{\beta_i}\left[\beta_i^{\frac{2}{n}} - \beta_i^{\frac{n+1}{n}}\right] = \beta_{cr}^{-\frac{1}{n}}\sqrt{\frac{n+1}{n-1}}\,d\tau. \quad (29)$$

The left side of Eq. (29) is a binomial differential, which can be integrated for n = 1.4. Performing the integration, we obtain

$$\tau = \beta_{cr}^{\frac{1}{n}} \sqrt{\frac{n-1}{n+1}} \left[\left(\frac{7}{5} D_1^5 + \frac{14}{3} D_1^3 + 7D \right) - \right.$$
$$\left. - \left(\frac{7}{5} D_2^5 + \frac{14}{3} D_1^3 + 7D_2 \right) \right], \tag{30}$$

where

$$D_1 = \sqrt{\left(1 - \beta_1^{\frac{2}{7}}\right) / \beta_1^{\frac{2}{7}}}, \quad D_2 = \sqrt{\left(1 - \beta_2^{\frac{2}{7}}\right) / \beta_2^{\frac{2}{7}}}$$

and

$$n = 1.4; \quad \beta_{cr}^{\frac{1}{n}} \sqrt{(n-1)/(n+1)} = 0.257.$$

For subcritical flow and adiabatic variation of the state of gas in the vessel Eq. (19) becomes

$$dt = \frac{V (p_i/p_{i0})^{\frac{1-3k'}{2k'}} d(p_i/p_{i0})}{\alpha f k' \left(\frac{2gnRT_{i0}}{n+1} \beta^{\frac{2}{n}} \right)^{1/2} \left(\left(\frac{p_a}{p_i} \right)^{\frac{2}{n}} - \left(\frac{p_a}{p_i} \right)^{\frac{n+1}{n}} \right)^{1/2}} \times$$
$$\times \frac{1}{\left(\frac{n+1}{n-1} \right)^{1/2} \beta_{cr}^{-\frac{1}{n}}}, \tag{31}$$

which can be written in the form

$$\frac{k'}{\beta_{i0}^{\frac{1-k'}{2k'}}} d\tau = \beta_{cr}^{\frac{1}{n}} \sqrt{\frac{n-1}{n+1}} \left(\beta_i^{\frac{2}{n}} - \beta_i^{\frac{n+1}{n}} \right)^{-\frac{1}{2}} \beta_i^{-\frac{k'+1}{2k'}} d\beta_i. \tag{32}$$

On the right side of the equation there is a differential binomial, the condition of integrability of which is satisfied when n = k' = 1.4. For k' = 1.4, Eq. (32) can be written in the form

$$1.4 \beta_{i0}^{0,143} d\tau = 0.257 \left(1 - \beta_i^{\frac{2}{7}}\right)^{-\frac{1}{2}} \beta_i^{-\frac{11}{7}} d\beta_i. \tag{33}$$

After integration we obtain

$$1.4\beta_{i0}^{0,143}\tau = -1.8 \left[\left(\frac{\sqrt{1-\beta_1^{\frac{2}{7}}}}{4\beta_1^{\frac{4}{7}}} + \frac{3\sqrt{1-\beta_1^{\frac{2}{7}}}}{8\beta_1^{\frac{2}{7}}} + \right.\right.$$
$$\left. + \frac{3}{16} \ln \frac{1+\sqrt{1-\beta_1^{\frac{2}{7}}}}{1-\sqrt{1-\beta_1^{\frac{2}{7}}}} \right) - \left(\frac{\sqrt{1-\beta_2^{\frac{2}{7}}}}{4\beta_2^{\frac{4}{7}}} + \right. \tag{34}$$
$$\left.\left. + \frac{3\sqrt{1-\beta_2^{\frac{2}{7}}}}{8\beta_2^{\frac{2}{7}}} + \frac{3}{16} \ln \frac{1+\sqrt{1-\beta_2^{\frac{2}{7}}}}{1-\sqrt{1-\beta_2^{\frac{2}{7}}}} \right) \right].$$

Figure 3 shows the curves of pressure variation for subcritical emptying. The curves were calculated from formulas (30) and (34). Here, as in the case of filling, emptying is more rapid for adiabatic variation of the state of the gas in the vessel. The emptying curve for the adiabatic process was calculated for $\beta_{i0} = 0.53$.

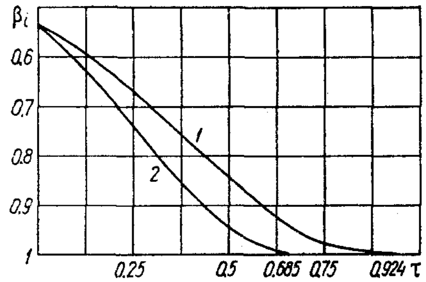

Fig. 3. Graphs for determining the emptying time in the case of subcritical flow: 1) k = 1, n = 1.4; 2) k = n = 1.4, $\beta_{i0} = 0.53$.

Filling of vessel of limited capacity and variable volume for variable cross section of opening. We have examined filling and emptying in the case of constant volume. In the case of variable volume, from the condition of conservation of the amount of gas we have

$$dG = G_{\text{sec}} dt = d\left(\frac{V}{v_i}\right). \tag{35}$$

Here V and v_i are variables expressed by the following relations:

$$V = V_0(1+\mu), \quad \mu = V_i/V_0, \quad V_i = V - V_0,$$
$$v_i = v_{i0}\left(\frac{p_{i0}}{p_i}\right)^{\frac{1}{k'}} = \frac{RT_{i0}}{p_{i0}}\left(\frac{p_{i0}}{p_i}\right)^{\frac{1}{k'}} = \frac{p_{i0}^{\frac{1-k'}{k'}}}{p_i^{\frac{1}{k'}}} RT_{i0}. \tag{36}$$

Substituting (36) in (35), we obtain

$$\frac{G_{\text{sec}} RT_{i0}}{V_0} dt\, p_{i0}^{\frac{1-k'}{k'}} = d\left[p_i^{\frac{1}{k'}}(1+\mu)\right]. \tag{37}$$

For supercritical flow Eq. (37) has the form

$$\alpha f p_0 \sqrt{\frac{1}{RT_0}} \sqrt{\frac{2gn}{n+1}\beta_{\text{cr}}^{\frac{2}{n}} RT_{i0}\, dt\, p_i^{\frac{1-k'}{k'}} V_0^{-1}} = d\left[p_i^{\frac{1}{n}}(1+\mu)\right]. \tag{38}$$

Consider the case where the initial temperature of the gas in the vessel is equal to the temperature of the gas in the pressure source ($T_0 = T_{i0}$), which is valid enough for many devices. For isothermal variation of the state of the gas we obtain

$$\alpha f p_i \sqrt{T_0} \sqrt{\frac{2gn}{n+1}\beta_{\text{cr}}^{\frac{2}{n}} v_0^{-1}}\, dt = d\left[p_i(1+\mu)\right]. \tag{39}$$

We replace the variable section of the opening by the relation

$$f = f_{\max}\psi(t), \quad \psi(t) = f/f_{\max}. \tag{40}$$

The value of the function $\psi(t)$ is given either by a system of differential equations or, as in the case of machines and engines, by a kinematic relationship – the distribution diagram expressing the relationship between the cross section of the openings and the coordinate of the piston.

In view of (40) and (8), Eq. (39) can be written in the form

$$\psi(\tau)\, d\tau = d[\beta_i(1+\mu)]. \tag{41}$$

Integrating (41) with the initial conditions

$$\tau = 0, \quad \mu = \mu_{\text{in}}, \quad \beta_i = \beta_{\text{in}},$$

we have

$$\beta_i = \frac{\beta_{in}(1+\mu_{in})}{1+\mu} + \int_0^{\tau} \frac{\psi(\tau)\,d\tau}{1+\mu}. \tag{42}$$

The first term of the obtained expression gives the change of gas pressure in the vessel when its volume changes, while the second gives the increase in pressure due to filling.

If the function $\psi(\tau)$ is known, the equation can be integrated by the numerical method for both supercritical and subcritical filling.

For subcritical filling the second term is determined from Fig. 1. In this case filling is examined in an isolated short interval of time τ_i, during which the values of $\psi(\tau_i)$ and $(1+\mu_i)$ are known and constant.

Once the pressure increment in τ_i has been determined from Fig. 1, it must be multiplied by the ratio $\psi(\tau_i)/(1 + \mu_i)$. The value obtained will represent the pressure increment in the interval of time considered.

Emptying of vessel of limited capacity and variable volume for variable cross section of opening. When the cross section of the opening and the volume of the vessel are both variable, the loss of gas from the vessel and the parameters of the gas in the vessel are related as follows:

$$-dG = G_{sec}dt = d\left(\frac{V}{v_i}\right) \tag{43}$$

or for supercritical flow

$$\alpha f\sqrt{\frac{p_i}{v_i}}\sqrt{\frac{2gn}{n+1}\beta_{cr}^{\frac{2}{n}}}\,dt = -d\left[\frac{V_0(1+\mu)}{v_{i0}(p_{i0}/p_i)^{1/k'}}\right] = -\frac{V_0}{v_{i0}}\,d\left[(1+\mu)\left(\frac{p_i}{p_{i0}}\right)^{\frac{1}{k'}}\right]. \tag{44}$$

In this case the values of p_i and v_i are the variable values of the pressure and specific volume of the gas in the vessel.

Using the relations

$$v_i = v_{i0}\,(p_{i0}/p_i)^{\frac{1}{k'}}, \quad p_i = p_{i0}\,(v_{i0}/v_i)^{k'},$$

we find

$$\frac{p_i}{v_i} = \frac{p_{i0}}{v_{i0}}\left(\frac{p_i}{p_{i0}}\right)^{\frac{k'+1}{k'}}. \tag{45}$$

Now

$$\alpha f\sqrt{\frac{p_{i0}}{v_{i0}}\left(\frac{p_i}{p_{i0}}\right)^{\frac{k'+1}{k'}}}\sqrt{\frac{2gn}{n+1}\beta_{cr}^{\frac{2}{n}}}\,dt = -\frac{V}{v_{i0}}\,d\left[(1+\mu)\left(\frac{p_i}{p_{i0}}\right)^{\frac{1}{k'}}\right]. \tag{46}$$

Using (8) and (40), we obtain

$$\beta_i^{\frac{k'+1}{2k'}}\,\psi(\tau)\,d\tau = -d\left[\beta_i^{\frac{1}{k'}}(1+\mu)\right]. \tag{47}$$

For isothermal variation of the state of gas in the vessel ($k' = 1$), after division of (47) by $\beta_i(1 + \mu)$, we have

$$\frac{d[\beta_i(1+\mu)]}{\beta_i(1+\mu)} = -\frac{\psi(\tau)\,d\tau}{1+\mu}. \tag{48}$$

With initial conditions $\tau = 0$, $\mu = \mu_{in}$, $\beta_i = \beta_{in}$ the integral of (48) will be

$$\frac{\beta_i (1+\mu)}{\beta_{in} (1+\mu_{in})} = \exp - \int_0^\tau \frac{\psi(\tau)\, d\tau}{1+\mu}, \tag{49}$$

and, hence, the variable pressure in the vessel is

$$\beta_i = \frac{\beta_{in} (1+\mu_{in})}{1+\mu} \exp - \int_0^\tau \frac{\psi(\tau)\, d\tau}{1+\mu}. \tag{50}$$

In the case of subcritical flow, when $\beta_i > \beta_{cr}$, the value of β_i is calculated as follows.

We write (50) in the form

$$\beta_i = \frac{\beta_{in} (1+\mu_{in})}{1+\mu} - \frac{\beta_{in} (1+\mu_{in})}{1+\mu} \left(1 - \exp - \int_0^\tau \frac{\psi(\tau)\, d\tau}{1+\mu}\right). \tag{51}$$

In this case the second term gives the pressure drop in the vessel due to emptying. For subcritical flow the value of the second term must be replaced by the value determined from Fig. 2 for the small interval of time during which $\psi(\tau_i)/(1+\mu_{i-1})$ can be regarded as constant.

NOTATION

V, V_0 – variable and initial values of volume of vessel; f, f_{max} – variable and maximum cross sections of opening; p_{i0}, v_{i0}, T_{i0} and p_i, v_i, T_i – initial and variable values of pressure, specific volume, and temperature of gas in vessel; p_a, T_a – pressure and temperature of medium into which vessel empties; p_0, v_0, and T_0 – pressure, specific volume, and temperature of source from which vessel is filled (assumed constant); G – gas flow per second; g – acceleration of gravity; n – index of polytropic process of variation of state of gas in stream; k' – index of adiabatic variation of state of gas in vessel; t – time; α – flow coefficient.

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